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OPTIMAL STRUCTURAL DESIGN OF REINFORCED CONCRETE STRUCTURES – REVIEW OF EXISTING SOLUTIONS CONSIDERING APPLICABILITY ASPECT

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ABSTRACT

During the past two decades, rapid advances in information technology have improved the accuracy and capabilities of optimization techniques. Unfortunately, great number of optimization methods in civil engineering has not found implementation in practice, mainly because the problems were treated only from mathematical point of view, disregarding applicability of obtained solutions in reality. Since the aim of any optimization method is developing methodology that would successfully imitate human reasoning, it is necessary to develop adequate approach that would obtain realistic and applicable solutions. This paper provides basic facts of the optimal reinforced structures design, problem classification, mathematic definition and applicability aspect, as well as preview of methods and references available in the literature.

Key words: *structural design, reinforced concrete structures, optimization, applicability*

INTRODUCTION

Common practise in structural design of the reinforced concrete structures includes determining cross-sectional dimensions and reinforcement that would meet the requirements proscribed by a given code of practise considering primarily strength and serviceability, as well as other imposed demands that result from the environment, architectural requirements etc. If the requirements are not met, than the cross-sectional dimensions and/or amount of the reinforcement have to be iteratively modified until all the required criteria are satisfied. In engineering practise, i.e. in reality, this iterative process is usually carried out without deeper consideration of prices of concrete, steel, formwork and human labour. Therefore, it is obvious that the practicing engineers need an efficient designing method that would give results which would be not only satisfying considering given legal standards but also considering optimality criteria.

The search for an effective and applicable method for optimal design of the concrete structures is not a new subject, but the most of developed applications and procedures were aimed at finding optimum (minimal) weight of a structure, although decision making process is usually, if not always, aimed at minimal price. Material and labour costs are important issues in design and construction of the reinforced concrete structures, as well as the applicability of obtained solution in practice, i.e. at the building site.

Rapid development of the information technology enabled researchers to develop innovative methods of design by including the optimality aspect more thoroughly and to include more realistic requirements and optimality criteria. The first variable studied in optimal structural design of the reinforced concrete structures is the cross-sectional shape of members that the structure is composed of. Theshapes are usually selected from a small list of available sections limited by form-work features and moreover the economical aspect that usually leads to mostly uniform forms withinone building. This reduction of the searched space therefore enables this part of design to berationally dealt with by the size optimization methods.

When the shape is determined and fixed, thesecond and possibly the most challenging task is the placement of reinforcing bars withinconcrete members, often called detailing. From the optimization point of view, thistask generally belongs to the field of topology optimization, where the number of bars, theirshape and material and even their mutual space position are searched for. The type and formof a chosen parameterization of the shape will determine the computational complexity of a given problem. Although this solution is the most straightforward one in terms of bothanalysis and the design phase, it is obvious that this approach is unmanageable with proper, often very complex mathematical models and adequate computational resources.

The great majority of approaches presented in the literature is formulated and focused on optimizing the cross-sectional dimensions and total quantity of reinforcement without further analysis of the reinforcement pattern and possibility of its proper placing and fixing at the building site. However, proper reinforcementdesign should include the specification of many details beyondthe determination of area of steel such as the selection ofbar diameters and the number of bars, the longitudinal distribution of group of bars that have the same size andlength, the positioning of bars at critical sections, determination of curtailment points, specification of the size andspacing of stirrups. As the cost, flexural strength and shearstrength of a member is a function of both the reinforcement detailing and dimensions of the member, detailingof reinforcement should also be considered during the optimization process.Asa consequence, this methods are adequate for analysis but not for practical use.

The aim of this paper is to present structural, mathematical and practical aspect of optimal reinforced structures structural design as a starting point for further researches, as well as to provide concise and clear overview of existing solutions and their basic features in order to enable other researchers to easily find adequate benchmark problems and to develop proper criteria for optimality assessment.Proposed review of works published until 2013 includes basic assumptions of given methods and codes of practice they were based on, which are the main starting points in finding relevant references for the research.

PROBLEM CLASSIFICATION

Structural optimization techniques and methods have been developed simultaneously all across the world under different names and terminologies, but the most general and the most complete classification of these approaches is probably the one proposed by Prof. Grant Steven [1]:

a) Topology optimization

Topology optimization is the most generalized type of problems because the task is to find a structure without knowing its final formbeforehand. The only known data are the environment (spans and loads), the optimality criteria (usually the lowest weight or price) and the constraints considering allowable stresses and displacements. This class of problems is quite common in mechanical engineering, while their representative in the field of civil engineering is determining optimal structural system for bridge or roof structure as well as the truss structures in which the position of joints and members is not know in advance. In this case, the objective is usually minimization of amount of material subjected tostructural requirements.

b) Shape optimization

In this form of optimization the topology of structure is a priori known in general but there can be some feature and/or detail of the structure that should be improved. Therefore the objective is usually to find the best shape that will result in the most suitable stressdistribution. Parameters of shapes are dimensions of the optimized parts or a set of variablesdescribing the shape. Examples for the reinforced concrete structures can be finding the proper shape of holes within plate members, the shape of a beam with holes and the optimal shape of pre-cast retaining walls.From the mathematical point of view, two types of variables can be introduced – continuous and discreteones.

c) Size optimization

This is the most common type of structural optimization problems because a structure is a priori defined by a set of sizes, dimensions or cross-sections that should be combined in order to achieve the desired optimality criteria. Within this area twomain groups of structures can be distinguished – discrete and continuum structures.

In discrete structures, all variables values are selected from the pre-defined discreteadmissible set. Therefore, this type of problems is characteristic for the steel structures, especially the trusses, while reinforced concrete structures usually belong to the second group.

Continuum structures include beam-like structures defined by continuous variables which are not known in a priori, in contrast to the previous case. The basic exampleis a beam with moments of inertia defined as a continuous variable. All reinforced concrete optimization tasks, where the area of reinforcing steel is an unknown belong to this group.

d) Topography optimization

The main task inthis class of problems form is to determine the proper shape for shell, membrane or tent like structures. This is the least investigated part of structural optimization, especially in the area of reinforced concrete structures.

Each of the abovementioned types of problems can be solved witha distinct optimization strategy chosen in accordance with specific practical features and mathematical formulation of a given particular problem. Naturally, solving real-world problems usually demand combining these approaches because real structures never can be observed as strictly mathematical tasks. Therefore, an expert knowledge and engineering awareness of applicability of obtained solution in reality, i.e. at the building site, should also be included in creating appropriate mathematical model for particular problem.

MATHEMATICAL FORMULATION

If the optimal design of a given reinforced structure is cost-oriented, i.e. if the aim is to minimize the total price of a structure, than the objective function can be defined as:

$$F(x) = V_c P_c + W_s P_s + A_f P_f \qquad (1)$$

where*Vc*is the volume of concrete, *Ws*is weight of steel, *Af*is total area of formworkand *Pc*, *Ps*and *Pf*are unit price of concrete per m^3, of steel per kg and of formwork per m^2, respectively. Prices of the materials include material, fabrication and labor.Total amount of concrete and formwork can be calculated according to the obtained cross-sectional dimensions and span, while total amount of the steel can be calculated after adopting the final reinforcement pattern and depends on diameters and lengths of chosen bars and stirrups.Constraints in this optimization problem are based on geometry, serviceability and durability requirements proscribed by a given code of practice.

Since the structural geometry (spans and supports positions), material properties and prices and loads (except self-weight) are usually predefined in the designing process, variables in this problem are the cross-sectional dimensions, i.e. width *b* and depth *h*. Basically, there is no need to include the total steel area in critical cross sections in the variables because it can be calculated according to a given code of practice. However, this aspect of design should not be totally neglected since the complete solution of the problem should also include details about bars diameters and placing scheme, which can be quite a demanding task, especially considering the great variety of possible patterns and possibility of combining bars with different diameters or forming bundles consisting of same or different bars.

PRACTICAL ASPECT OF SOLUTION

After calculating required amount of the reinforcement for a given cross section, a designer is supposed to choose proper combination of reinforcing bars which would have the total area as close as possible to the calculated one, and to specify their exact positions in a cross section in accordance with rules and requirements given in a code of practice. Having in mind that reinforcement bars come in more than ten different diameters, this task is not as easy as it is usually considered.Although codes of practice can vary more or less between different countries, but they all generally come down to the same set of requirementsbecause what is obligated in one country usually is accepted as a rule of thumb in another and vice versa.In general, if bars with different diameters are used, greater diameter should be placed closer to the bottom edge and sides, i.e. closer to the concrete surface, and total area of bars in the lower row should be greater than or equal to the area of bars in the upper row. Combinations of significantly different diameters should be avoided and therefore the difference between the largest diameter and the smallest one should be limited by the maximal acceptable value, usually 5–6 cm or three bar sizes. The adequate concrete mixture pouring and vibrating should be allowed and enabled by defining minimal clear horizontal and vertical spacing between the bars, which should not exceed code specified value and the maximum bar diameter, Figure1.

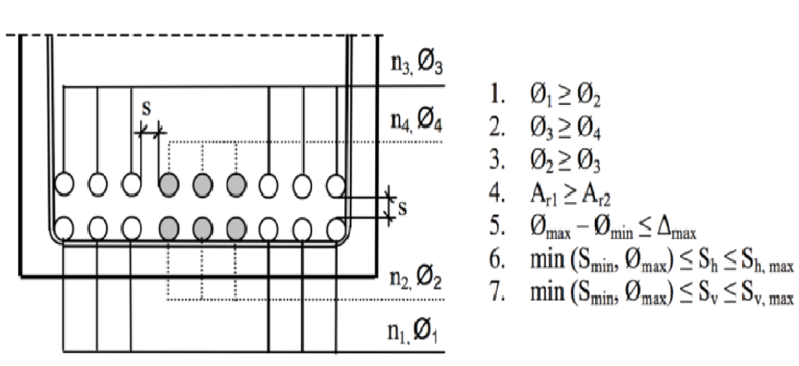

Figure 1 Typical reinforcement template

PROPOSED SOLUTIONS

Until the information technology was not developed enough to support very complex calculus models and procedures, problem of optimal reinforced structures design was usually solved by considering only basic variables such as cross-sectional dimensions and total amount of the reinforcement, while the problem of reinforcement bars placing within a concrete members (often called detailing) remained almost untouched, or was avoided by introducing too generalized assumptions.

Numerous researchers have investigated possibilities in optimal design of reinforced concrete girders and structures. Friel [2] derived an equation for determining optimal ratio of steel to total concrete area in a singly reinforced beam, while Chou [3]used Lagrange multiplier method for minimizing total cost of the T-shaped beam. Kirsch [4]presented iterative procedure in three levels of optimization for minimizing the cost of continuous girders with rectangular cross section, in which the total amount of the reinforcement is minimized at the first level, cross-sectional dimension are minimized at the second level, while the third level of optimization iz minimizing the design moments. Lakshmanan and Parameswaran [5]derived a formula for direct determining of optimal span to cross-sectional depth ratio so the iterative trial and error procedure can be avoided, while Prakash et al. [6] based their cost-minimization method on Lagrangian and simplex methods. Kanagasundaram and Karihaloo [7,8] introduced thecrushing strength of concrete as an additional variable along with cross-sectional dimensions and steel ratio to optimize the cost of simply supported and multi-span beamswith rectangular and T-sections using sequential linear programming and convex programming.Chakrabarty [9,10] presented cost-optimization method for rectangular beams using the geometric programming andNewton–Rapson method, while Al-salloum and Siddiqi [11]proposed optimal design of singly reinforced rectangular beams by taking the derivatives of the augmented Lagrangian function with respectto the area of steel reinforcement.Coello et al. [12] proposed the cost optimal design of singly reinforced rectangularbeam using Genetic Algorithms by considering cross-sectional dimensions and the reinforcement area asvariables. More detailed overview of literature on cost-optimization of reinforced concrete structures up to 1998 can be found in [13].

One of the first papers that deal with reinforcement placing details was presented by Koumousis andArsenis [14]. This method is based on multi-criterion optimization using Genetic Algorithmsfor finding a compromise between minimum weight, maximum uniformity and the minimum number of bars for a group ofmembers. After that, researchers have started to introduce reinforcement detailing data as variables in optimization methods, usually by using one of two basic approaches. In the first one, reinforcement spacing demands are included into calculus as constraints, while the other one uses previously developed data base of possible reinforcement patterns. Constraints in the first approach are based on maximum allowable number of reinforcement layers (usually one or two) and maximum allowable number of bars per layer (usually up to four or five). The second approach is in fact simplification of the first one because the data-base of allowable reinforcement patterns is developed by introducing the same limitations and demands proposed by a given code of practice.

Review of most important works in this field in the last fifteen years, including corresponding codes of practice and basic assumptions, is presented in Table 1. It can be observed that the main problem in comparing efficiency and applicability of different approaches is the fact that they are based on different codes of practice, i.e. on different reinforcement placing rules and restrictions. Because of that, and as opposite of the steel structures, there is no standard benchmark problems for testing a given method so the parametric sensitivity analysis is the only available tool for applicability assessment.

The other problem, and the more substantial one, is the great variety of different basic assumptions such as maximal allowed number of rows and number of bars per row. For example, limiting the number of bars per row on four or five is acceptable for cross sections with width up to 35 cm, while there is no reason to use such restriction for wider cross sections. Besides that, limitation of maximally one row of the reinforcement has no practical excuse, especially when dealing with narrow but tall cross sections.

Even the one of the most advanced approaches, proposed by Govindaraj and Ramasamy [15,16], has its limitations. Although based on the most relaxed constraints, allowing as much as three different bar diameters in the same cross section, this method uses previously developed data base of possible reinforcement patterns is based on assumption that the number of rows is limited to three and the number of bars per row is limited to five. Insofar, the only approach without any a priori adopted limitation beside the ones given in the Eurocode 2 is [17].

Table 1 Literature preview from 1998. to 2013.

Author	Code of practice	Basic assumptions
Koumousis & Arsenis [14]	Greek Code 1991	Maximum one row with not more than for bars of the same diameter.
Rajeev&Krishnamoorthy [18]	Indian Standard Code of Practice	Data base with 14 possible reinforcement patterns.
Matouš et al. [19] Lepš & Šejnoha[20]	EC2	Maximum 3 rows, maximum 31 bars per row, same diameters.
Camp et al. [21]	ACI99	Data base, maximum one row with maximum 4 bars, same diameters.
Lee & Ahn [22]	ACI99	Data base, maximum 2 rows with maximum 4 bars, same diameters.
Ferreira & Barros [23]	EC2	Only total steel area is considered.
Praščević [24]	PBAB87	Only total steel area is considered.
Yokota at al. [25]	Not specified	One row, number of bars between 3 and 10.
Barros at al. [26]	EC2	Only total steel area is considered.
Sahab et al. [27,28]	British Standard BS8110	Only columns are considered, one bar in each corner.
Guerra & Kiousis [29]	ACI05	Only total steel area is considered.
Govindaraj&Ramasamy [15,16]	Indian Standard Code of Practice	Data base, maximum 3 rows with maximum 5 bars per row, maximum 3differentdiameters.
Kwak & Kim [30,31]	Korean Code	Data base, maximum 2 rows, maximum 5 bars, same diameters.
Perera & Vique [32]	ACI05 + EC2	Only total steel area is considered.
Alqedra et al. [33]	ACI08	Number of bars between 4 and 12, same diameters.
Kaveh and Sabzi [34]	ACI08	Data base, maximum 2 rows with maximum 6 bars, same diameters.
Barros et al. [35]	EC2	Only total steel area is considered.
Bekdas & Nigdeli [36]	ACI2005	Maximum 2 rows with maximum 5 bars, same diameters.
Jahjouh et al. [37]	ACI 2008	Maximum 8 bars, same diameters, detailed pattern is not considered.
Yousif & Najem [38]	ACI 2008	3 data bases: 2 rows with a) same diameters, b) diferent diameters, c) both a) and b)
Milajić et al. [17]	EC2	No a priori assumptions.

CONCLUSION

The search for an effective and applicable method for optimal design of the concrete structures is not a new subject, but the great majority of procedures that can be found in the literature consider this problem only as the mathematical one, regardless of applicability of obtained solutions in practise, i.e.

in design of the real structures. Another difficulty is the fact that proposed methods are based on different assumptions and codes of practice, so there are no universal criteria and standard benchmark problems, such as in case of optimal design of steel structures. Because of that, researchers have to find and analyse numerous references and sources in order to find adequate ones for comparison and assessment of their methods results.

Purpose of this paper is an attempt to abridge a gap between theory and practise in the field of optimal design of the reinforced concrete structures by emphasizing importance of assessing obtained solutions from the practical point of view.The second part of the paper provides concise overview of existing solutions up to 2013 in order to enable researchers to find the adequate comparison criteria and benchmark problems for their solutions of the problem.

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